Causal Inference on Spatio-temporal Data

I Some Basics

"standard" causal modelling:

- variables $X_1, \dots, X_p$ either $X_i \in \mathbb{R}$ or $X_i \in \mathbb{N}$
- base model class = SCMs

$$X_i := f_i(X_{pa(i)}, U_i)$$

- $X_{pa(i)}$ = causal parents
- f_i = causal mechanisms
- U_i = noise terms
  ↳ often assumed independent
  = no confounding

Typical objects of interest:

- $P(X_1, \dots, X_p)$ observational distribution
- $P(\dots \mid do(X_i := x_i))$ interventional distributions
- $\frac{\partial}{\partial a} \mathbb{E}[X_j \mid do(X_i := a)]$ causal effects (direct / total)
- $P^{CF}(\dots \mid X^{obs} = \underline{x}, do(X_i^{CF} = x_i))$ counterfactual distributions

-
$$\begin{tikzcd} X_1 \arrow[r] \arrow[d] & X_2 \arrow[d] \\ X_3 \arrow[r] & X_4 \end{tikzcd}$$

causal graph

II Time series

$$X_1 = (X_1^t)_{t\in\mathbb{Z}},\quad X_2 = (X_2^t)_{t\in\mathbb{Z}},\quad \dots,\quad X_p = (X_p^t)_{t\in\mathbb{Z}}$$

Possible to sample many times from each series?

Yes

Can use the basic setting with variables

$$(X_i^t)_{(i,t)\in[p]\times\mathbb{Z}_{\geq 0}}$$

$$+\ (i,t)\in pa(j,t') \Rightarrow t \leq t'$$

No

Need some type of stationarity assumption to generate samples:

Relationship of X_i^t and $X_j^{t'}$ only depends on $|t-t'|$ (and i,j)

$$\begin{tikzcd}
X_1^{t-\tau_{max}} \arrow[d] & \dots & X_1^{t-1} \arrow[d] \arrow[r] & X_1^t \arrow[d] \\
X_2^{t-\tau_{max}} & \dots & X_2^{t-1} \arrow[r] & X_2^t \\
X_3^{t-\tau_{max}} & \dots & X_3^{t-1} \arrow[r] \arrow[ur] & X_3^t
\end{tikzcd}$$

Summary Graph

$$\begin{tikzcd}
X_1 \arrow[loop above] \arrow[d, bend left] \\
X_2 \arrow[loop right] \\
X_3 \arrow[loop right] \arrow[u, bend left]
\end{tikzcd}$$

Many causal discovery methods / effect estimation techniques in this setting
↳ PCMCI+ ...

In practice often deal with linear models

$$\underline{X}^t = \sum_{s=0}^{\tau_{max}} A_s \underline{X}^{t-s} + \underline{U}^t$$

with effect matrices $\underline{A} = (A_0 \dots A_{\tau_{max}})$
↑ special conditions on contemporaneous matrix

Can be recast as convolution model

$$\underline{X}^t = \underline{A} * \begin{pmatrix} \underline{X}^t \\ \vdots \\ \underline{X}^{t-\tau_{max}} \end{pmatrix} + \underline{U}^t$$

or $$\underline{X}^t = (I - A_0)^{-1} \, A' * \begin{pmatrix} \underline{X}^{t-1} \\ \vdots \\ \underline{X}^{t-\tau_{max}} \end{pmatrix} + \underline{U}^t$$

→ increasingly: methods in continuous setting.

III Spatiotemporal setting

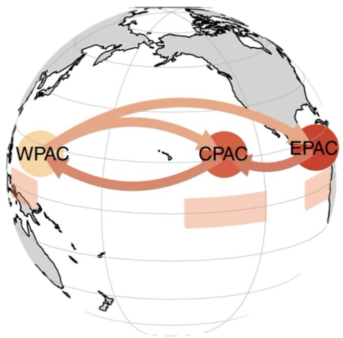
Objects of interest: spatiotemporal fields

$$X_1 = (X_1^{s,t})_{t \in \mathbb{Z}, \, s \in S^1}, \dots, X_p = (X_p^{s,t})_{t \in \mathbb{Z}, \, s \in S^p}$$

spatial domain

Are the domains the same?

no ↙ ↘ yes



- emergent problem
- local structure less important

Goal: • extract causality relevant information for global interactions

"classical" geostatistics

spatio-temporal Durbin model in econometrics

diffusion models

- local dynamics matter a lot

Requires notion of spatial stationarity,

e.g. interaction of $X_i^{s,t}$ and $X_j^{s',t'}$

depend only on

$|s-s'|$ and $|t-t'|$

and there is some maximal speed of transmission.

What we looked at last time was even more local

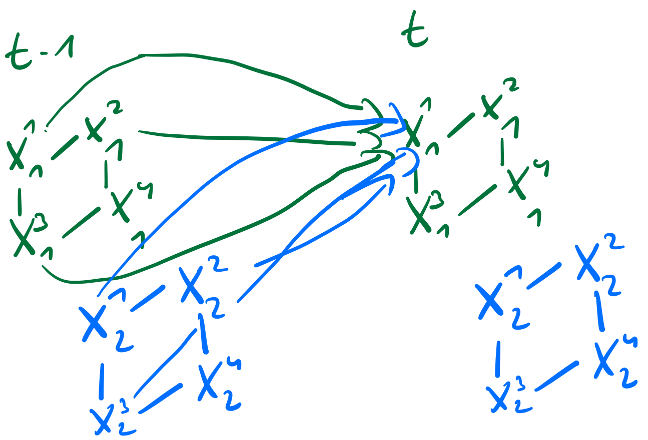
$$Y^{s,t} := f(X^{s,t}, H^{s,t}, \varepsilon^{s,t})$$

"no spillover"

A natural model to look at from a causal perspective are spatio-temporal convolution models

$$\underline{X}^{s,t} = \sum_{t'=0}^{\tau_{max}} \sum_{\substack{s' \text{ s.t.} \\ \frac{|s'-s|}{|t'-t|} < \tau_{max}}} A^{s',t'} \, \underline{X}^{s-s',t-t'} + \underline{U}^{s,t}$$

$$= A * \left(\underline{X}^{s',t'}\right)_{s',t'} + U^{s,t}$$



→ also transferable to continuous domains

Problems for causal discovery

- Conditional independence testing:

  Large conditioning sets reduce effective sample size

  Localization may help a little bit

- Assumptions may be even harder to argue for / against
- The system may have different components
  spatio-temporal fields / time or spatially invariant variables / regime variables
- In practice, applied researchers seem to work more with the potential outcome framework but under strong unconfoundedness assumptions.

ORIGINAL ARTICLE

Causal inference with spatio-temporal data: Estimating the effects of airstrikes on insurgent violence in Iraq

Georgia Papadogeorgou[1] | Kosuke Imai[2] | Jason Lyall[3] | Fan Li[4]